

SUSY Splits, But Then Returns

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Abstract

We study the phenomenon of accidental or “emergent” supersymmetry within gauge theory and connect it to the scenarios of Split Supersymmetry and Higgs compositeness. Combining these elements leads to a significant refinement and extension of the proposal of Partial Supersymmetry, in which supersymmetry is broken at very high energies but with a remnant surviving to the weak scale. The Hierarchy Problem is then solved by a non-trivial partnership between supersymmetry and compositeness, giving a promising approach for reconciling Higgs naturalness with the wealth of precision experimental data. We discuss aspects of this scenario from the AdS/CFT dual viewpoint of higher-dimensional warped compactification. It is argued that string theory constructions with high scale supersymmetry breaking which realize warped/composite solutions to the Hierarchy Problem may well be accompanied by some or all of the features described. The central phenomenological considerations and expectations are discussed, with more detailed modelling within warped effective field theory reserved for future work.

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1 Introduction

In the last decade, models of warped compactification have led to significant progress in understanding how non-supersymmetric dynamics can generate the electroweak and flavor hierarchies observed in particle physics. See Refs. [1] [2] for reviews and extensive references. These hierarchies ultimately derive from the redshift or “warp factor” that varies exponentially across extra-dimensional space. While the original Randall-Sundrum I (RS1) [3] braneworld model used such redshifts to solve the electroweak Hierarchy Problem, realistic modern variants with the Standard Model (SM) propagating in five-dimensional (5D) “bulk” spacetime also describe flavor hierarchies in terms of warped extra-dimensional wavefunction overlaps. The most spectacular (but challenging) experimental prediction of such models is the production of Kaluza-Klein (KK) excitations of gravitons and SM particles at several TeV (as reviewed in Ref. [2]).

Via the AdS/CFT correspondence [4] these warped models can be seen as effective descriptions of purely 4D theories [5] [6] [7], which however involve strong-coupling physics. The extra-dimensional warping that solves the hierarchy problem is dual to TeV-scale Higgs compositeness. Weak-coupling in the warped description reflects a $1/N_{color}$ -type expansion of the 4D strong dynamics. From the 4D perspective one must posit (i) a weakly-coupled gauge theory sector of elementary particles, (ii) a strongly-coupled matter sector also charged under (i), with a mass gap Λ_{comp} , below which lie a finite number of light composites and above which the dynamics is approximately conformally invariant, and (iii) a finite list of local operators of the strong conformal field theory (CFT) needed to couple the two sectors. If one further assumes that all other minimal color-singlet CFT operators¹ have large scaling dimensions, greater than some $\Delta \gg 1$, then $1/\Delta$ emerges as a new expansion parameter for the system. The combined $1/\Delta$ - and $1/N$ -expansion is precisely a weakly-coupled 5D warped effective field theory on an RS1 background. More complex situations can also arise with even higher dimensional warped descriptions (famously the case in $\mathcal{N} = 4$ supersymmetric Yang-Mills [4]), or without any higher-dimensional effective field theory regime. In this sense, 5D warped models are conjectured minimal realizations of (i – iii), but they display many qualitative features that appear to be robustly general.

Warped models represent a powerful and comprehensive approach to the hierarchy problem and other challenges of particle physics, providing an attractive alternative to the paradigm of Weak Scale Supersymmetry (SUSY). But there are two important regards in which they are not completely satisfactory. The first is theoretical, in that higher-dimensional effective field theory is non-renormalizable and warping pushes this feature to relatively low energies. Even though many leading effects, even quantum effects, are calculable in such a framework, they ultimately rely on the *existence* of a UV completion. At present, superstring theory provides the main hope for finding such UV completions of minimal or non-minimal warped models. But since we are trying to explore a non-SUSY solution to the hierarchy problem, superstring constructions must contend with breaking SUSY at very high scales while still retaining control of the rather complex dynamics. As a result this subject is still in its infancy, with constructions aiming at proofs of principle rather than full realism [8] [9].

¹For instance, these are single-trace operators in theories with only color-adjoint “gluons”.

The second issue is phenomenological. Any resolution of the hierarchy problem by new TeV-scale physics must explain why that new physics has not been seen in direct searches or through its virtual effects in precision tests. Warped models are in a sense highly successful in this regard, but still imperfect, in that evading precision tests require pushing the new physics (KK excitations dual to Λ_{comp} -scale composites) to several TeV, leading to percent-level or worse residual fine-tuning to account for the smaller electroweak scale. Analogous tensions are seen in other approaches to the hierarchy problem and are collectively known as the Little Hierarchy Problem.

The purpose of the present paper is to study the joint role SUSY might play in each of these two issues, without however stealing the entire spotlight. The plot is as follows. It may well be true that SUSY is a fundamental symmetry of spacetime that appears in string theory UV completions of warped models of particle physics, but with very large SUSY-breaking scale. The subtleties of SUSY breaking and transmission in warped settings is therefore important to understand. One such subtlety is that despite high-scale SUSY-breaking, vestiges of SUSY can be “accidentally” redshifted (warped) down from high scales to low scales $\sim \Lambda_{comp}$. This can be seen in both string and effective field theory examples [10] [11] [12] [13] [8] [9]. From the dual 4D viewpoint this corresponds to SUSY being an accidental or “emergent” symmetry of the strongly-coupled sector. This vestigial or “partial SUSY” is insufficient by itself to solve the “Big” Hierarchy Problem, which is instead solved by warping/compositeness as in non-SUSY models, but it has the power to neatly address the Little Hierarchy Problem by stabilizing a modestly larger Λ_{comp} /weak-scale hierarchy than naturally allowed in non-SUSY models. Such a mechanism is not merely a theoretical nicety: the physics that resolves the Little Hierarchy Problem will dominate particle physics experiments in the years to come, in particular the Large Hadron Collider (LHC).

We build on the work of Ref. [12], which (in 4D language) proposed the co-existence of emergent SUSY of the strong sector composites with the absence of SUSY in the weakly-coupled sector. They argued that this Partial SUSY could be protected at strong coupling in a manner dual to the extra-dimensional separation of the two sectors in a warped compactification. With the Higgs bosons and top quarks among the SUSY composites, partial SUSY would in turn protect the “little hierarchy” between Λ_{comp} and the weak scale. But while top quark loops give the largest Higgs radiative corrections that destabilize the weak scale, gauge loops are not far behind. We show that there is a more powerful extension of partial SUSY that follows by replacing the completely non-SUSY weakly-coupled sector by a weakly-coupled sector of Split-SUSY [14] [15] [16] type, in particular containing light gauginos. This extension then suppresses all the dominant Higgs radiative corrections in the little hierarchy.² A new analysis is given of the requisite SUSY cancellations in gauge loops, taking into account that the gauginos and gauge particles are elementary, unlike the Higgs-top-stop system which consists entirely of low-scale composites.

We also carefully discuss instabilities in the proposal of partial SUSY which arise from

² It is important to distinguish our proposal from that of section 3.5 of Ref. [14], which also combines Split SUSY with a strong CFT sector at the TeV scale. The central difference is that the CFT of Ref. [14] does not have any (accidental) SUSY, and there is no extra suppression of the dominant Higgs corrections beyond that offered by Higgs compositeness.

AdS-tachyons which are scalar superpartners of gauge fields in the warped bulk. While such tachyons do not signify instabilities of infinite AdS spacetime, they do represent potential instabilities of warped compactifications after high-scale SUSY breaking [8]. Indeed, attempts at string UV-completion must inevitably deal with some version of this issue, and it explains the challenge in finding such constructions. Here we study the issues in their most minimal setting. We will show how such instabilities can be avoided, which in realistic settings results in new gauge particles at collider energies.

In this way, we will describe a partnership in establishing the electroweak hierarchy, between SUSY and warping/compositeness. SUSY may be in charge at the very highest scales, with warping/compositeness then generating and protecting a significant hierarchy, and then partial SUSY returning at the TeV scale to generate a little hierarchy. Phenomenologically, the lowest rung of this ladder has similar collider implications to More Minimal SUSY [17], with the addition of new TeV-scale gauge particles, then followed by multi-TeV KK-excitations/massive composites. Most of our discussion in this paper will be given in 4D language, in which the basic “grammar” of the story can be worked out using the power of the renormalization group (RG) to connect disparate scales. This will lay the foundation for Ref. [18], in which minimal warped models will be built and more quantitatively analysed.

The paper is organised as follows. In Section 2, we discuss the phenomenon of accidental SUSY within gauge theory and its connection to strongly coupled matter. In Section 3, we review a simple structure of UV SUSY breaking compatible with Split SUSY. In Section 4, we review how a mass gap, Λ_{comp} , can arise in the IR of a strong CFT and then study how effective the associated physics is at mediating SUSY breaking effects to the IR. We study how light composites, in particular scalars, of the accidentally supersymmetric strong dynamics feel UV SUSY breaking. The discussion in this section is restricted to the weakly-coupled sector being (accidentally) pure SUSY Yang-Mills theory. Section 5 discusses an accidentally SUSY strong sector coupled to a Split SUSY weakly-coupled sector. This is our final target, yielding our improved version of partial SUSY. We focus on how naturalness of composite scalars works in this setting. In Section 6, we give a lightning CFT-dual review of realistic non-SUSY warped models. In Section 7, we then incorporate partial SUSY and focus on several of the key issues for realistic model-building. In Section 8, we briefly discuss the experimental implications of our scenario.

This work grew out of an initial collaboration with Thomas Kramer, investigating the possibility of realistic partial SUSY [19]. However, the proposal to fuse partial SUSY with Split SUSY is new to the present paper, as are a number of lesser issues.

2 Accidental Supersymmetry within Gauge Theory

IR fixed points readily appear in the RG flows of quantum effective field theories. Consider such a fixed point with a robust basin of attraction. That is, after imposing some symmetries in the UV, for a non-fine-tuned set of UV couplings the theory flows towards the fixed point, at least over a large range of energies. The CFT describing the fixed point may possess greater symmetry than is present in the UV, in which case the enhanced symmetry is referred to as “accidental” or “emergent”. The fixed point is at best a limit of the RG flow, so accidental

symmetries are never exact but become better approximations the further the IR running towards the fixed point. Here, we study the case of accidental supersymmetry [20] in the context of gauge theories with perturbatively weak gauge coupling. Refs. [13] [8] discussed the case where there is no weakly-coupled gauge theory outside the strong dynamics.³

2.1 Accidental SUSY Yang-Mills theory

Pure SUSY Yang-Mills (SYM) offers the only completely weak-coupling example of accidental SUSY in gauge theory [20].⁴ Suppose that in the far UV, SUSY is either non-existent or badly broken, so that at lower energies we have a non-SUSY gauge theory consisting of a gauge field and a Weyl fermion, λ^a , in the adjoint representation of the gauge group. Let us assume that the UV dynamics respects the chiral symmetry $\lambda \rightarrow e^{i\theta} \lambda$ (more precisely, a non-anomalous discrete subgroup of this symmetry) so that a fermion mass term is forbidden. Then the effective gauge theory takes the form

$$\mathcal{L}_{eff}(M) = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\lambda}iD.\sigma\lambda + \frac{\text{higher dimension operators}}{M^k}, \quad (1)$$

where M is the scale of massive physics we have integrated out. Since the higher-dimension operators are not constrained to respect SUSY, at scales just below M the theory is far from supersymmetric. But in the far IR the theory is well approximated by just the *renormalizable* gauge and chirally invariant interactions,

$$\mathcal{L}_{eff}(\mu \ll M) = -\frac{1}{4}F_{\mu\nu}^2 + \bar{\lambda}iD.\sigma\lambda, \quad (2)$$

which is accidentally supersymmetric. In this way, SUSY has emerged or re-emerged in the IR. Identifying the IR theory as approximate SYM, λ is identified with the gaugino and its chiral symmetry with R -symmetry.

Note that for any particular starting UV couplings, there is a limit to how far we can run towards supersymmetry in the IR, given by the mass gap or confinement scale, “ Λ_{QCD} ”, but this can naturally be $\ll M$. Physically, the spectrum of “hadrons” will not be exactly supersymmetric, but will have small SUSY-breaking splittings of order Λ_{QCD}/M , due to the residual sensitivity to non-renormalizable interactions.

2.2 Charged matter via strong coupling

Let us try to generalize to accidental SUSY gauge theories with charged matter. It is immediately obvious that examples do not exist at weak coupling. Any such example beyond SYM would result in chiral supermultiplets (in $\mathcal{N} = 1$ SUSY language) which contain scalars. Whatever the quantum numbers of such scalars ϕ , with the exception of SUSY which we take

³In Ref. [13], weakly-coupled gauge fields emerge in the IR as composites of the strong dynamics.

⁴ Ref. [20] *derives* the particle content of SYM in the IR using strong dynamics in the UV. In this subsection we simply assume the right particle content and review how SUSY couplings emerge in the IR. In this way we avoid the strong coupling regime of Ref. [20].

to be broken or non-existent in the UV, we cannot naturally forbid a completely symmetric mass term,

$$\mathcal{L}_{eff}(M) \ni -M^2|\phi|^2, \quad (3)$$

and without fine-tuning ϕ will not survive in the IR.

But there is an important loop-hole to this argument at strong coupling, since large anomalous dimensions might give the completely symmetric $|\phi|^2$ operator a true scaling dimension > 4 , thereby making it an *irrelevant* SUSY-breaking perturbation, that flows towards zero in the IR. Note that we are assuming the gauge dynamics discussed above is *weakly-coupled*, and that the strong coupling is due to some other interactions of the charged matter (possibly gauge interactions associated with a different gauge group). At first sight, the possibility that strong interactions can make a highly-relevant SUSY-breaking mass term into an irrelevant effect might seem a far-fetched hope, in any case untestable in the face of the difficulties of strong coupling calculations. But remarkably, strongly-coupled large- N_{color} $\mathcal{N} = 4$ supersymmetric Yang-Mills theory demonstrates precisely this mechanism [8]! The gauge scalars of this theory, have a completely gauge- and R-symmetric bilinear operator $|\phi|^2$, which has a scaling dimension of order $N_{color} \gg 4$ (at strong coupling), thereby making it an irrelevant SUSY-breaking deformation of the theory. This is established by examining the AdS/CFT dual string description.

Having seen this strong-coupling “miracle” occur under theoretical control, we should consider that it might well be a robust phenomenon among strongly-coupled theories, and might play an important role in the real world. Let us therefore further explore the possibility of accidental SUSY in weakly-coupled gauge theory with strongly-coupled matter.

2.3 Minimal set of relevant and marginal couplings

We consider the possibility of a strongly-coupled matter sector that flows towards a supersymmetric fixed point, described by a superconformal field theory (SCFT), weakly coupled to a set of gauge fields and “accidental gauginos” (in the sense described in subsection 2.1). Let us be maximally optimistic and assume that the fixed point is as attractive as possible in the IR. That is, we assume the SCFT has the fewest possible relevant scalar operators that might appear in the interactions of the UV theory and push it away from the SUSY fixed point in the IR. If there are no highly relevant scalar operators we will have accidental SUSY. We want to check whether this is consistent with the general structure of a SCFT of gauge-charged matter.

Every CFT comes with an energy-momentum tensor $T_{\mu\nu}$ with scaling dimension 4. Because it is conserved and traceless, it contains no scalar component. In a SCFT, there must also be a vector current related to $T_{\mu\nu}$ by SUSY, but again it is conserved and contains no scalar component, and is just the symmetry current of the superconformal R -symmetry. Thus the absolutely minimal structure of a SCFT does not imply on general grounds the existence of scalar operators with $\mathcal{O}(1)$ scaling dimensions [13].

In addition, we want the SCFT matter to be weakly gauged by some external gauge fields and “gauginos”. This means that prior to being gauged, the SCFT possesses a non- R symmetry, with conserved currents J_μ^a , again without any scalar component. But now SUSY

necessitates scalar operators D^a in the same supermultiplet as J_μ^a . These are nothing but “ D – terms”, which are the familiar simple “squark” bilinears in weakly coupled matter sectors, but at strong coupling such identifications become less useful and we work directly with the D^a operators and their symmetry and scaling properties. Conserved currents J_μ have protected scaling dimension 3, and SUSY then implies that the D^a have protected scaling dimension 2. We must therefore worry about relevant deformations of the UV theory of the form

$$\mathcal{L}(M) \ni \mathcal{L}_{SCFT} + M^2 D, \quad (4)$$

which would prevent it from flowing towards the supersymmetric fixed point, without fine-tuning. Clearly however, for non-abelian gauge symmetry, this is forbidden by gauge invariance itself. So we must only consider abelian factors of the gauge group. Even here, exact discrete symmetries such as charge-conjugation invariance can forbid such couplings linear in D ’s. We assume for the rest of this section that such protective symmetries are in place. In connecting to the real world this provides a tight model-building requirement as we discuss in Section 7.

Thus the contrast between strongly-coupled and weakly-coupled SCFT limits of charged matter is this. Both contain dimension-2 scalar operators corresponding to potentially relevant SUSY-breaking deformations of the SCFT, but it is only weakly coupled SCFTs that necessarily possess dimension-2 scalar operators invariant under all (non-SUSY) symmetries, that cannot naturally be forbidden from appearing in the UV.

Let us now assemble the minimal set of couplings of the SCFT matter and the external gauge fields, A_μ^a , and gauginos, λ^a . We will do this without regard to SUSY, just imposing gauge invariance (and any protective non-SUSY symmetries for abelian gauge groups and massless gauginos), and keeping all couplings which are relevant or marginal in the IR. We will then study the extent to which these couplings flow towards SUSY relationships in the IR. The most obvious of these is the gauge coupling, g , of gauge fields to themselves (if non-abelian) and also to SCFT matter. The SCFT supermultiplet containing the conserved current J_μ^a , and the scalar operator D^a , also contains a fermionic operator Ψ_J^a with protected scaling dimension 5/2. These can couple gauge-invariantly and marginally to the λ^a .

So far we have only considered couplings linear in the conserved currents of the SCFT or in operators in the same supermultiplets. But we must also consider couplings involving Lorentz-invariant *products* of such operators. In general however, the relevance of such operator products is difficult to establish since their scaling dimensions need not be algebraically related to those of their factors. We will therefore specialize to the case of strongly-coupled SCFTs with a large- N expansion, in which case the scaling dimension of a product is given simply by the sum of the dimensions of its factors, up to $1/N$ corrections. We will denote the associated parameter “ N ” from now on as “ N_{CFT} ” in order to be clear that it does not refer to the size of the weakly coupled gauge sector external to the CFT, which is taken to be parametrically smaller. In such an expansion, we see that the only scalar product operator of the SCFT that need concern us is the approximately marginal (and completely symmetric) product $D^a D^a$. The effective Lagrangian describing the RG flow is therefore given simply

by

$$\mathcal{L}(\mu) = -\frac{1}{4}F_{\mu\nu}^a{}^2 + \bar{\lambda}D.\sigma\lambda + \mathcal{L}_{SCFT} + gA_\mu^a J_a^\mu + \tilde{g}\lambda_a \Psi_J^a - \frac{1}{2}g_D^2 D^a D^a. \quad (5)$$

Here the term “ $gA_\mu^a J_a^\mu$ ” is schematic. It means that the SCFT is gauged by the external gauge group, with coupling g . In general this is a non-linear coupling to A_μ determined by gauge invariance, but at linearized order it takes the above form. Note that we are insisting on the UV chiral symmetry of the “gaugino” λ , protecting against a mass term $\lambda\lambda$. In this way, our RG flow is parametrized by just three (approximately) marginal couplings, g, \tilde{g}, g_D^2 .

2.4 RG in the large- N_{CFT} expansion

Let us start by working out the RG equations for g, \tilde{g}, g_D^2 , to one-loop order in these weak couplings, and to leading order in the large- N_{CFT} expansion of the strong interactions of the SCFT sector.

The gauge coupling running is dominated by the large- N_{CFT} matter in vacuum polarization,

$$\frac{dg^2}{d\ln\mu} = \frac{\mathcal{O}(N_{CFT})}{16\pi^2}g^4. \quad (6)$$

The fermionic operator Ψ_J in the \tilde{g} coupling is not renormalized purely by the SCFT dynamics since it is related to the conserved current J_μ by SUSY. It can be corrected by further g, \tilde{g}, g_D^2 couplings, but without N_{CFT} -enhancement. However, \tilde{g} can be renormalized by wavefunction renormalization of the gaugino λ by the SCFT, which is N_{CFT} -enhanced. Note this involves two more \tilde{g} couplings, so that

$$\frac{d\tilde{g}^2}{d\ln\mu} = \frac{\mathcal{O}(N_{CFT})}{16\pi^2}\tilde{g}^4. \quad (7)$$

Similarly, in g_D^2 renormalization, the factors of D^a are not renormalized purely by the SCFT dynamics at the fixed point, D^a being related by SUSY to the conserved current, and corrections to this from external interactions are not N_{CFT} -enhanced. Instead, the large- N_{CFT} enhanced renormalization comes at order $(g_D^2)^2$, from contractions of the form $g_D^4 D^a \langle D^a D^b \rangle D^b$. Therefore,

$$\frac{dg_D^2}{d\ln\mu} = \frac{\mathcal{O}(N_{CFT})}{16\pi^2}g_D^4. \quad (8)$$

Note that if we had exact SUSY, it would require

$$\begin{aligned} \mathcal{L}(\mu) &= \mathcal{L}_{SCFT} + \int d^2\theta \mathcal{W}_\alpha^a \mathcal{W}_a^\alpha + g \int d^4\theta V_{gauge} J_{CFT} \\ &= \mathcal{L}_{SCFT} - \frac{1}{4}F_{\mu\nu}^a{}^2 + \bar{\lambda}D.\sigma\lambda + gA_\mu^a J_a^\mu + g\lambda_a \Psi_J^a - \frac{1}{2}g^2 D^a D_a, \end{aligned} \quad (9)$$

where in the second line we have integrated out the scalar auxiliary superpartner of the gauge field.⁵ That is, exact SUSY would require

$$\tilde{g}^2 = g_D^2 = g^2. \quad (10)$$

⁵Off-shell, this auxiliary field is distinct from the composite operator of the matter sector, D^a , but becomes equal to it after solving its equation of motion.

We do not insist on SUSY in the UV, but we want to see if the theory flows towards this relation in the IR. But the RG flow must preserve the SUSY relations if they were satisfied in the UV. This tells us that the $\mathcal{O}(N_{CFT})$ coefficients of each of the RG equations for g^2, \tilde{g}^2, g_D^2 is the *same*, so that

$$\frac{d1/g^2}{d \ln \mu} = \frac{d1/\tilde{g}^2}{d \ln \mu} = \frac{d1/g_D^2}{d \ln \mu} \equiv -b_{CFT} \sim \frac{\mathcal{O}(N_{CFT})}{16\pi^2}. \quad (11)$$

Of course, these same RG equations govern the flow of couplings even when they are non-supersymmetrically related. Let us assume that at some UV scale, m_0 , we start with three unrelated non-SUSY couplings, $g_0, \tilde{g}_0, g_{D0}^2$. In the IR, these flow to

$$g^2 = \frac{g_0^2}{1 + b_{CFT} g_0^2 \ln(m_0/\mu)}; \quad \tilde{g}^2 = \frac{\tilde{g}_0^2}{1 + b_{CFT} \tilde{g}_0^2 \ln(m_0/\mu)}; \quad g_D^2 = \frac{g_{D0}^2}{1 + b_{CFT} g_{D0}^2 \ln(m_0/\mu)}. \quad (12)$$

Working to first order in the splittings of the g 's,

$$\begin{aligned} \frac{\Delta \tilde{g}^2}{g^2} &\equiv \frac{\tilde{g}^2 - g^2}{g^2} \approx \frac{g^2}{g_0^2} \frac{\Delta \tilde{g}_0^2}{g_0^2} \\ \frac{\Delta g_D^2}{g^2} &\equiv \frac{g_D^2 - g^2}{g^2} \approx \frac{g^2}{g_0^2} \frac{\Delta g_{D0}^2}{g_0^2}. \end{aligned} \quad (13)$$

Since the gauge coupling running is dominated by SCFT matter, it is IR free, $b_{CFT} > 0$, and g falls with RG scale μ . Therefore, we see that there is a focussing effect for the splittings in the IR. Even one hundred percent differences among the $g_0^2, \tilde{g}_0^2, g_{D0}^2$, can still evolve to small fractional differences in the IR. This is the central realization of accidental SUSY within gauge theory.

2.5 Important RG corrections

The above analysis is formally leading in the large- N_{CFT} limit, but it misses a qualitatively important correction, namely the effects which are leading for small g . If one flows sufficiently far into the IR such effects will always dominate given the IR-free nature of the gauge coupling, driven by the large amount of SCFT matter.

This is not an issue for the β -functions of g^2, \tilde{g}^2 , which necessarily start at $\mathcal{O}(g^4)$ (where we are taking any of the g^2, \tilde{g}^2, g_D^2 to be roughly comparable) because renormalization occurs either via self-energy corrections to gauge fields or gauginos, in which the claim is obvious, or via vertex corrections to the $\lambda\psi_J$ vertex. In this latter case, an $\mathcal{O}(g^2)$ correction to the ψ_J operator is required for non-trivial renormalization because the supersymmetric strong dynamics at the fixed point alone does not renormalize this “superpartner” of the conserved current J_μ .

However, the g_D^2 β -function does get an $\mathcal{O}(g^2)$ correction. First of all there is a possible multiplicative renormalization of g_D^2 , due to strong dynamics dressing of the $D^a D^a$ operator. But it is straightforward to see that this is a $1/N_{CFT}$ -suppressed effect since it involves connecting two separate CFT-color singlet factors. Gauge field and gaugino exchanges alone

cannot additively renormalize g_D^2 , because their spin forbids their producing the correct scalar-scalar coupling of strong operators, and because such exchanges are not 1PI. Both of these deficiencies however are corrected by further strong dynamics dressing, but again only in a $1/N_{CFT}$ -suppressed manner. Consequently, the RG equation for g_D^2 is modified to the form

$$\frac{dg_D^2}{d\ln\mu} = b_{CFT}g_D^4 + \mathcal{O}(1/N_{CFT})g_D^2 + \mathcal{O}(1/N_{CFT})g^2 + \mathcal{O}(1/N_{CFT})\tilde{g}^2. \quad (14)$$

Again, the various $\mathcal{O}(1/N_{CFT})$ coefficients must be related in such a way as to preserve the SUSY relationship $g^2 = \tilde{g}^2 = g_D^2$ if it holds in the UV. Working to first order in $\Delta g_D^2, \Delta\tilde{g}^2$, this implies

$$\frac{d\Delta g_D^2}{d\ln\mu} = 2b_{CFT}g^2\Delta g_D^2 - \gamma_D\Delta g_D^2 - \tilde{\gamma}\Delta\tilde{g}^2, \quad (15)$$

where we have new constants,

$$\gamma_D, \tilde{\gamma} \sim \mathcal{O}(1/N_{CFT}). \quad (16)$$

We see that the γ terms can become more important in the IR because of IR freedom of the gauge coupling and the scaling of these terms as $\mathcal{O}(g^2)$ rather than $\mathcal{O}(g^4)$. In more detail, we can simplify by noting that the flow of $\Delta\tilde{g}^2$ is unaffected and therefore our earlier derivation is still valid, namely that it actually scales as $\mathcal{O}(g^4)$ in the IR. Thus we can consistently neglect its feedback into Δg_D^2 via $\tilde{\gamma}$ in the IR flow. The dominant danger to our large- N_{CFT} analysis comes from γ_D . Keeping just this new effect, the full set of RG equations is given by,

$$\begin{aligned} \frac{dg^2}{d\ln\mu} &= b_{CFT}g^4 \\ \frac{d\Delta\tilde{g}^2}{d\ln\mu} &= 2b_{CFT}g^2\Delta\tilde{g}^2 \\ \frac{d\Delta g_D^2}{d\ln\mu} &= (2b_{CFT}g^2 - \gamma_D)\Delta g_D^2. \end{aligned} \quad (17)$$

These corrected RG equations are still straightforward to solve,

$$\begin{aligned} g^2 &= \frac{g_0^2}{1 + b_{CFT}g_0^2 \ln(m_0/\mu)} \\ \frac{\Delta\tilde{g}^2}{g^2} &= \frac{g^2}{g_0^2} \frac{\Delta\tilde{g}_0^2}{g_0^2} \\ \frac{\Delta g_D^2}{g^2} &= \frac{g^2}{g_0^2} \frac{\Delta g_{D0}^2}{g_0^2} \left(\frac{m_0}{\mu}\right)^{\gamma_D}. \end{aligned} \quad (18)$$

If $\gamma_D > 0$ we cannot trust these solutions arbitrarily into the IR. At some point, even though $\gamma_D \sim \mathcal{O}(1/N_{CFT})$, power law growth of Δg_D^2 in μ will take over, and it will be necessary to treat Δg_D^2 beyond first order. In the remainder of the paper we will take care that we are in the linear regime.

We have just discussed subleading effects in large- N_{CFT} that are of order $\mathcal{O}(g^2)$. There remains one last class of effects in the RG equations that appears at one-loop order in the g^2, \tilde{g}^2, g_D^2 deviations from the exact fixed point SCFT dynamics, namely all the subleading effects in large- N_{CFT} that are of order $\mathcal{O}(g^4)$. The simplest of these is just the running contribution to the gauge coupling due to gauge fields and gauginos themselves when the external gauge group is non-abelian,

$$\frac{d1/g^2}{d\ln\mu} = -b_{CFT} + b_{SYM}. \quad (19)$$

For example, for an $SU(n)$ external gauge group (n formally smaller than N_{CFT}), $b_{SYM} = 3n/(8\pi^2)$. But the new complication at this order is that the linearized RG equations for Δg_D^2 and $\Delta \tilde{g}^2$ will in general be coupled to each other, and consequently harder to solve. However, these corrections are truly subdominant to the effects calculated above. In this paper, we will neglect them.

The power of minimal warped compactification models related to this scenario by AdS/CFT, is that the entire set of RG corrections discussed above is calculable in terms of just the one strong interaction parameter, b_{CFT} , and group theory considerations. We will do a more complete analysis of these corrections in Ref.[18]. Below we shall just illustrate the value-added in minimal warped models by showing how γ_D is determined in terms of b_{CFT} . In particular we verify that $\gamma_D > 0$, so that it indeed signifies a (slightly) relevant effect in the IR.

2.6 γ_D in minimal warped compactification

Let us translate some of the issues surrounding the SCFT operators D^a and $D^a D^a$ into the context of warped compactifications, using AdS/CFT. The fact that the CFT is gauged by an external gauge group, means in isolation it must contain corresponding conserved currents J_μ^a . The dual of these currents and the external gauge fields is given by 5D gauge fields propagating on a 5D warped RS background. SUSY at the fixed point translates into minimal SUSY in the 5D bulk, which means that the bulk minimally also contains 5D gauginos and 5D gauge scalars. It is these gauge scalars which are dual to the D^a . The scaling dimension 2 of the D^a matches the AdS-SUSY constraint that the 5D gauge scalars are AdS tachyons saturating the Breitenlohner-Freedman bound [21]. This set-up is our minimal bulk system. 5D gravity, dual to the CFT energy-momentum tensor, can naturally play a subdominant role (for sufficiently large 5D Planck mass). We are imagining that SUSY is violently broken or absent on the UV boundary, but that the rest of the dynamics is supersymmetric.

The bulk 5D SUSY Yang-Mills dynamics is controlled by a single parameter, the 5D gauge coupling, g_5^2 , which can be expressed dimensionlessly, g_5^2/R_{AdS} , in terms of the AdS radius of curvature. We can identify it within the SCFT description by doing a tree-level matching

from 5D to the IR 4D gauge coupling below the KK scale:

$$\begin{aligned}\frac{1}{g^2} &= \frac{1}{g_0^2} + \frac{L}{g_5^2} \\ &= \frac{1}{g_0^2} + \frac{\ln(m_0/\Lambda_{comp})R_{AdS}}{g_5^2}.\end{aligned}\tag{20}$$

Here, $1/g_0^2$ is the coefficient of the UV boundary-localized kinetic term for the gauge fields, L is the length of the fifth dimension, m_0 is the UV scale and $\Lambda_{comp} = m_0 e^{-L/R_{AdS}}$ is the resulting warped down IR scale. AdS/CFT identifies this equation with the RG equation for the external gauge coupling at leading order in large N_{CFT} . We thereby arrive at the identification,

$$b_{CFT} \equiv \frac{R_{AdS}}{g_5^2}.\tag{21}$$

While AdS tachyons at or above the Breitenlohner-Freedman bound, such as our gauge scalars, do not represent an instability of AdS, they do represent a violent instability in an RS background if they are allowed tadpole couplings on the UV boundary, since these lead to scalar profiles that rapidly blow up towards the IR of the warped bulk. This is precisely why we needed to introduce protective symmetries in the UV. In the 5D picture these symmetries forbid one from writing or radiatively generating such a tadpole couplings in the UV. In the dual picture they protect against deformations of the SCFT linear in the D^a .

However, one must also consider quantum loops of the tachyons in the 5D theory, with effects analagous to Casimir energy in a compact space. Such loops allow tachyon-pairs to propagate into the IR of the warped bulk, dual to the effects of deforming the SCFT by $D^a D^a$. These loops cannot be forbidden by any symmetry. SUSY can enforce their cancellation but we have taken it to be maximally broken on the UV boundary, say by writing UV-boundary localized mass terms for the 5D gauge scalars. We are interested in seeing how efficiently this UV SUSY breaking propagates into the IR as a result of the tachyon loops. At leading order the tachyons are free particles in the bulk, dual to the leading large- N_{CFT} approximation, in which the operator $D^a D^a$ simply has twice the dimension of D^a . That would make it a marginal dimension-4 deformation. However, minimally, the gauge scalars (for non-abelian gauge group) will exchange gauge bosons in the bulk. It is precisely this effect that is dual to the γ_D correction in the SCFT.

We can compute it efficiently (following a similar strategy to Refs. [22]) by studying the SCFT correlator

$$\begin{aligned}\langle D^a(0)D^a(x)D^b(x')D^c(x'') \rangle &\ni_{x \rightarrow 0} \frac{\langle (D^a D^a)(0)D^b(x')D^c(x'') \rangle}{|x|^{\gamma_D}} \\ &\approx_{\gamma_D \rightarrow 0} \langle (D^a D^a)(0)D^b(x')D^c(x'') \rangle \times (1 - \gamma_D \ln |x|).\end{aligned}\tag{22}$$

The first line is part of an OPE, with the exponent of x following by dimensional analysis using the scaling dimensions of all the local operators. The second line is expanded formally for small γ_D . We know that $g_5^2/R_{AdS} = 1/b_{CFT} \propto 1/N_{CFT}$, formally the same order as γ_D . Therefore perturbative AdS_5 calculations of the above correlator in g_5^2 are automatically

perturbative expansions in γ_D . No finite set of Yang-Mills Feynman diagrams in AdS will reproduce the first line, but the leading orders will give us the second line and we can then “RG improve” the result to get the first line. In fact all we want to extract is γ_D , which requires us to compare the small x asymptotics of the original correlator at leading (zeroth order in g_5^2) and next-to-leading (order g_5^2). This translates into the AdS correlator of four gauge scalars by the standard AdS/CFT prescription, with leading order corresponding to free field theory approximation for the scalars, and next-to-leading order corresponding to single-gauge-boson exchange between the scalars in 5D. The first of these is straightforward, while the latter can be extracted from the work of Ref. [23]. Assembling the result gives

$$\gamma_D = \frac{ng_5^2}{12\pi^2 R_{AdS}} \equiv \frac{n}{12\pi^2 b_{CFT}}, \quad (23)$$

if the gauge group is $SU(n)$, and zero if it is abelian.

We will use this relation in Section 7 in making some numerical estimates.

3 R-Symmetric UV SUSY breaking

We now want to move to considering SUSY as a fundamental symmetry of the far UV, but a badly broken one. We can capture the high scale breaking of fundamental SUSY, or indeed its complete absence, in a simple way. We assume that SUSY breaking originates from some hidden sector dynamics with vacuum energy V_0 , and is communicated to our gauge and matter sectors by some massive physics of typical mass scale M . Integrating out this massive physics, and with sufficient non-SUSY symmetries in the UV (including the R -symmetries that protect against gaugino masses), V_0 is the only SUSY breaking “spurion” felt by the gauge and matter sectors, suppressed by appropriate powers of $1/M$. We thereby arrive at the estimates

$$\Delta\tilde{g}(M), \Delta g_D^2(M) \sim \frac{V_0}{M^4}. \quad (24)$$

We can use these as the initial conditions for our RG analysis, identifying $m_0 \equiv M$.

The complete absence of UV SUSY can be identified with the choice $V_0 \sim M^4$, but more generally we can consider $V_0 < M^4$, in which case we see that the initial conditions of our analysis can easily be highly supersymmetric, with further supersymmetric focussing in the IR. The SUSY breaking terms in the gauge and strongly-coupled matter sectors are “hard” because there are no highly relevant operators allowed, whose coefficients would be “soft terms”. Hard breaking of a symmetry can in general naturally be small when the symmetry breaking spurion is dimensionful, as our case illustrates. In a weakly coupled matter sector by contrast, one would naturally have soft scalar mass terms, $\sim V_0/M^2$, and SUSY in this sector would be badly broken at $\sqrt{V_0}/M$, which could easily be a very large scale.

If SUSY starts as a fundamental spacetime symmetry, we cannot ignore the effects of supergravity. Anomaly-mediated SUSY breaking (AMSB) [24] is a supergravity correction that typically introduces gaugino masses $\sim \frac{g^2}{16\pi^2} \sqrt{V_0}/M_{Pl}$, although in special circumstances this effect can be much smaller [10] [14] [16]. Even in the more general case the effect can

be subdominant to the effects we consider, by taking $M \ll M_{Pl}$. We will choose parameters in this regime when we give illustrative numerical estimates later. The gravitino mass is of order $\sqrt{V_0}/M_{Pl}$, which we will again use in our later estimates to determine the lightest stable SUSY particle (LSP).

4 Mass Gap in the Strong Sector

The treatment of accidental SUSY in gauge theory is both incomplete as is and not directly applicable to the real world. Our linearized treatment of Δg_D^2 is incompatible with flowing arbitrarily into the IR because of the blow-up due to $\gamma_D > 0$ in Eq. (18). Even if we could keep flowing into the IR, SCFT behavior is not compatible with the SM physics we eventually want to recover in the IR. Both issues can be resolved by arranging for the matter sector to be (approximately) superconformal down to a finite scale Λ_{comp} , at which the strong dynamics produces composite states. Let us first see the simplest way to do this.

4.1 Generating Λ_{comp}

The generation of a mass gap in the CFT sector is AdS/CFT-dual to “radius” stabilization in RS1, originally achieved by the Goldberger-Wise mechanism [25]. Here we will discuss a SUSY version of this mechanism [26] in the dual (S)CFT description, and then the effect of high-scale SUSY breaking. The discussion is similar but not identical to that of Ref. [13]. The starting point is to assume that the SCFT has an *approximate* moduli space. (For example, the moduli space could become exact only in the large- N_{CFT} limit.) We consider minimally a single SUSY chiral superfield modulus, ω . Conformal invariance is only intact at the origin of moduli space, while a non-trivial vacuum expectation value (VEV) $\langle \omega \rangle$ (spontaneously) breaks it. We must deviate from the exact SCFT in order to stabilize such a VEV $\langle \omega \rangle = \Lambda_{comp}$.

Let us introduce a new SCFT operator, a gauge-singlet chiral primary, \mathcal{O} , with scaling dimension $3 + \epsilon$, where $\epsilon \ll 1$ (in practice, $\epsilon \sim 1/10$). We can write a SUSY deformation of the SCFT in the UV,

$$\mathcal{L}(M) = \mathcal{L}_{SCFT} + c \int d^2\theta \frac{\mathcal{O}}{M^\epsilon} + \text{h.c.} \quad (25)$$

We can match its effects to the ω effective field theory below Λ_{comp} ,

$$\mathcal{L}_{eff} = \int d^4\theta |\omega|^2 + \int d^2\theta (\lambda \omega^3 + c\kappa \frac{\omega^{3+\epsilon}}{M^\epsilon}) + \text{h.c.}, \quad (26)$$

where κ represents a strong interaction matrix element arising in matching the deformation. The other terms are superconformally invariant. If $\lambda = 0$, ω would be an exact modulus of the SCFT, but we are taking it as only approximate, so λ is small but non-zero. Clearly, the SUSY vacuum satisfies

$$\Lambda_{comp} \equiv \langle \omega \rangle \sim (\lambda/c\kappa)^{1/\epsilon} M, \quad (27)$$

which can naturally account for a large hierarchy from M down to Λ_{comp} .

When we consider UV SUSY breaking there is a new subtlety, since now we can perturb the SCFT by the lowest component of \mathcal{O} , which we will denote by \mathcal{O}_\perp , rather than the highest component $\int d^2\theta \mathcal{O}$. Such a highly relevant deformation at maximal strength,

$$\mathcal{L}(m_0) = \mathcal{L}_{SCFT} + M^{1-\epsilon} \mathcal{O}_\perp + \text{h.c.}, \quad (28)$$

would completely destabilize the SCFT and accidental SUSY. However, our SUSY-breaking spurion analysis of the last section generalizes to this case, and gives

$$\mathcal{L}(M) = \mathcal{L}_{SCFT} + V_0/M^{3+\epsilon} \mathcal{O}_\perp + \text{h.c.} \quad (29)$$

The resulting soft scale of SUSY breaking, $(V_0/M^{3+\epsilon})^{1/(1-\epsilon)}$, can naturally be small compared to all other effects we study.

Alternatively, we might be interested in the case of no SUSY in the UV, which we explained is equivalent to $V_0 \sim M^4$. In this case, R -symmetry can be used instead to forbid the SUSY breaking coupling \mathcal{O}_\perp , while permitting the SUSY preserving one, $\int d^2\theta \mathcal{O}$. Since \mathcal{O} has scaling dimension $3+\epsilon$, it has superconformal R -charge $2+2\epsilon/3$, so even $\int d^2\theta \mathcal{O}$ breaks R -symmetry. We therefore assign its coefficient c a spurious R -charge of $-2\epsilon/3$. Similarly, the \mathcal{O}_\perp coupling must have spurious R -charge $-2-2\epsilon/3$, and therefore can naturally have strength $\sim c^{3/(\epsilon+1)}$. Even modestly small c is compatible with very strongly suppressed SUSY-breaking \mathcal{O}_\perp coupling.

In summary, we have a supersymmetric mechanism at hand that explains the emergence of the IR mass gap in the strong sector, Λ_{comp} . It requires a new supermultiplet of operators of the CFT with $\mathcal{O}(1)$ scaling dimensions. But it is natural for significant SUSY breaking not to be communicated via these operators. Closely analogous operators (or their AdS/CFT duals) appear in string constructions of warped compactifications compatible with high scale SUSY breaking. See the discussion of Ref. [9].

4.2 Composite Scalars of Accidental SUSY

Most of these composites of the strong dynamics will have masses set by Λ_{comp} , but a few may be light. The accidental SUSY means that these light composites come in supermultiplets, most simply chiral supermultiplets, Φ . These are in general charged under our original gauge group, since the SCFT was weakly gauged. Then the effective theory below Λ_{comp} is given by

$$\begin{aligned} \mathcal{L}_{eff} = & \int d^4\theta \Phi^\dagger e^{gV} \Phi + \int d^2\theta (\mathcal{W}_\alpha^2 + W_{eff}(\Phi)) + \text{h.c.} \\ & + \Delta \tilde{g} \sqrt{2} \phi^\dagger \lambda \psi_\phi + \text{h.c.} - \frac{\Delta g_D^2}{2} \left(\sum_\phi \phi^\dagger T^a \phi \right)^2. \end{aligned} \quad (30)$$

Here, the first line contains the SUSY approximation, including a superpotential coupling between composites and a SUSY D-term potential for composite scalars ϕ upon integrating out the gauge scalar auxiliary fields. The second line however contains the SUSY breaking

effects that have survived the RG flow, but they constitute hard breaking of SUSY, not soft terms. The first hard breaking is a non-SUSY correction to the gaugino coupling to the fermionic “current”, now composed of charged composites. The second term is a SUSY-breaking correction to the D -term potential for charged scalars, the SCFT D -term operators, D^a , matching onto the composite field D -terms, $\sum_\phi \phi^\dagger T^a \phi$.

Let us see what the impact of (approximate) accidental SUSY is on the natural scale of light composite scalar masses, m_ϕ . Even without SUSY, compositeness provides a natural mechanism for having light scalars. For example, even pure non-SUSY Yang-Mills theory produces light scalar glueballs with masses of order Λ_{QCD} , which can naturally be small. But this is the analog of $m_\phi \sim \Lambda_{comp}$. We are really inquiring about the natural hierarchy that can exist between Λ_{comp} and m_ϕ . Of course, the best known non-SUSY example is a composite Nambu-Goldstone boson, whose masslessness is protected by a spontaneously broken symmetry. But such a scalar has only derivative couplings. By contrast the accidental SUSY provides us with light scalars which can have non-derivative Yukawa couplings described by W_{eff} . These couplings do not impact scalar masses by the non-renormalization theorem. This is essentially the proposal of Ref. [12], and we have given it in general terms and have corrected it by including symmetry protection against terms linear in the D^a .

The couplings of the strong matter sector to the external gauge fields radiatively contributes to the masses of light scalar composites at one-loop order (just as QED contributes to the mass of the charged pion of QCD),

$$\Delta m_\phi^2 \sim \frac{g^2}{16\pi^2} \Lambda_{comp}^2, \quad (31)$$

the usual quadratic divergence of fundamental scalars replaced by Λ_{comp}^2 for composite scalars. If there were no gauginos, such contributions would set a limit on the natural hierarchy, $m_\phi \sim \frac{g}{4\pi} \Lambda_{comp}$. This is the usual behavior in composite Higgs models [27] [28].

But the presence of the gaugino means that there is a possible accidental SUSY cancellation among the gauge corrections to scalar masses. Crucially, this cancellation requires the right SUSY coupling of the gaugino to the composites, as well as the right SUSY D -term potential among composite scalars. The deviations are measured by the hard breaking coefficients $\Delta \tilde{g}^2, \Delta g_D^2$. As we have seen, without SUSY in the UV (or with badly broken SUSY in the UV) the hard breaking is big. But substantial running between m_0 and Λ_{comp} can cure that, so that net gauge radiative corrections to scalar masses are given dominantly by

$$\Delta m_\phi^2 \sim \frac{\Delta g^2}{16\pi^2} \Lambda_{comp}^2 \sim \frac{\Delta g_0^2}{g_0^2} \frac{g^2}{16\pi^2} \frac{g^2}{g_0^2} \left(\frac{m_0}{\Lambda_{comp}}\right)^{\gamma_D} \Lambda_{comp}^2. \quad (32)$$

Let us first consider that we start with order hundred percent UV breaking of SUSY relations among g, \tilde{g}, g_D^2 at m_0 , and ask what the best case is for accidental SUSY to extend the natural hierarchy between Λ_{comp} and m_ϕ . For this, we need to assume that γ_D is small enough (N_{CFT} is large enough) that $(\frac{m_0}{\Lambda_{comp}})^{\gamma_D}$ is still roughly of order one, and we need to take the gauge coupling to start near strong coupling, $g_0^2 \sim 16\pi^2$. We then see that the gauge radiative corrections cancel to within

$$\Delta m_\phi^2 \sim \left(\frac{g^2}{16\pi^2}\right)^2 \Lambda_{comp}^2. \quad (33)$$

This is a gain of an (IR) gauge loop factor relative to Eq. (31), without the gaugino. It translates to a theoretical maximum natural hierarchy

$$\frac{m_\phi}{\Lambda_{comp}} \sim \frac{g^2}{16\pi^2}. \quad (34)$$

This is comparable to the improvement in the natural scale separation in going from standard composite Higgs models to Little Higgs models [29].

In the absence of UV SUSY, accidental SUSY has bought two gains, one being the (maximally) extra gauge loop factor from cancellations of gauge radiative corrections, and the other that large composite Yukawa interactions do not impact scalar masses radiatively. Yet there is a limit to the natural hierarchy. In phenomenological terms, if one finds the light composite scalars then generic composites are at most an inverse-loop-factor away (up to possible fine-tuning).

However, if there is some degree of SUSY in the UV couplings $g_0, \tilde{g}_0, g_{D0}^2$, then this can extend the natural hierarchy further. From the perspective of this paper, this case of $V_0 \ll M^4$ is “too good”, in that there can be a very large separation between Λ_{comp} and m_ϕ , as seen by plugging Eq. (24) into Eq. (32). Then, accidental SUSY protects m_ϕ at the lowest energies, but the strong coupling to which it owes its existence is hidden at far higher energies, Λ_{comp} . Up to fine details (still worthy of study), the low-energy phenomenology resembles the standard SUSY protection of light scalars. The case of no UV SUSY (equivalent to $V_0 \sim M^4$), investigated above is more phenomenologically interesting precisely because Λ_{comp} and m_ϕ are naturally separated, but not arbitrarily so. Nevertheless we have discussed both cases for completeness, and as prerequisites for the next section.

5 Split SUSY meets Accidental SUSY

The most interesting interplay between fundamental SUSY, accidental SUSY, and compositeness occurs when there are two types of gauged matter sectors, one which is strongly coupled and flowing to a SCFT as discussed above, and the other which consists of the more familiar weakly coupled matter comprising some charged chiral supermultiplets. We follow the discussion of Section 3 and specialize to the case $V_0 \ll M^4$. This will result in the highly suppressed hard SUSY breaking discussed there in the gauge and strongly-coupled matter sectors, as well as soft mass terms, $m_0^2 \sim V_0/M^2$, for the weakly coupled matter. This means that above m_0 the gauge theory is highly supersymmetric, but below m_0 we must integrate out the massive scalars of the weakly-coupled chiral multiplets, leaving only their fermionic partners, ψ_{weak} . This large SUSY splitting of matter multiplets, but without gaugino masses, is the basic Split SUSY set-up, but here we also have the strongly-coupled matter sector.

The effective theory below m_0 in totality therefore does not even have supersymmetric particle content, let alone SUSY interactions:

$$\begin{aligned} \mathcal{L}(\mu < m_0) &= \mathcal{L}_{SCFT} + \int d^2\theta \, \mathcal{W}_\alpha^a \mathcal{W}_a^\alpha + g A_\mu^a J_a^\mu + \tilde{g} \lambda_a \Psi_J^a - \frac{1}{2} g_D D^a D^a, \\ &+ \bar{\psi}_{weak} i \gamma^\mu D_\mu \psi_{weak}. \end{aligned} \quad (35)$$

Nevertheless, the leading matching at the m_0 -threshold has g 's satisfying the SUSY relations

$$g_0^2 \approx \tilde{g}_0^2 \approx g_{D0}^2, \quad (36)$$

inherited from M , with $\mathcal{O}(V_0/M^4)$ corrections which we neglect.

Despite the new weakly-coupled matter content, the IR theory still only has relevant and marginal couplings g, \tilde{g}, g_D^2 . Let us denote the size of the weak matter sector by N_{weak} , and we assume that

$$N_{CFT} \gg N_{weak}. \quad (37)$$

Therefore to leading order in large- N_{CFT} the RG equations are unchanged. But again, there is a qualitatively important effect that is missed in this limit. The RG equations of the previous section preserved the SUSY relations of the g 's, if they were satisfied in the UV. Here, these relations are indeed well satisfied at m_0 , but the corrections to the RG flow due to the ψ_{weak} , which are subleading in N_{CFT} , no longer preserve the SUSY relations among the g 's. Since we are interested in SUSY cancellations in the radiative corrections of composite scalars, we must retain these SUSY-breaking RG effects.

Adding these corrections, the RG equations are given by

$$\begin{aligned} \frac{dg^2}{d \ln \mu} &= (b_{CFT} + b_{weak})g^4 \\ \frac{d\tilde{g}^2}{d \ln \mu} &= b_{CFT}\tilde{g}^4 \\ \frac{dg_D^2}{d \ln \mu} &= b_{CFT}g_D^4 - \gamma_D(g_D^2 - g^2), \end{aligned} \quad (38)$$

where the only terms subleading in N_{CFT} retained are those that push us away from the SUSY relations among the couplings. Here,

$$b_{weak} \sim \mathcal{O}(N_{weak})/16\pi^2 \quad (39)$$

describes the effect on gauge-coupling running due to charged ψ_{weak} loops. Without their superpartner scalars however, the weak sector has no other interaction than its gauge interactions. In particular it cannot contribute at one-loop order in the g 's to the running of \tilde{g}, g_D^2 .

We deduce the following linearized RG equations for splitting in the g 's,

$$\begin{aligned} \frac{d\Delta\tilde{g}^2}{d \ln \mu} &= 2b_{CFT}g^2\Delta\tilde{g}^2 - b_{weak}g^4 \\ \frac{d\Delta g_D^2}{d \ln \mu} &= (2b_{CFT}g^2 - \gamma_D)\Delta g_D^2 - b_{weak}g^4. \end{aligned} \quad (40)$$

These have solutions,

$$\begin{aligned} \frac{\Delta\tilde{g}^2}{g^2} &= \frac{g^2}{g_0^2} \frac{\Delta\tilde{g}_0^2}{g_0^2} + b_{weak}g^2 \ln(m_0/\mu) \\ \frac{\Delta g_D^2}{g^2} &= \frac{g^2}{g_0^2} \frac{\Delta g_{D0}^2}{g_0^2} \left(\frac{m_0}{\mu}\right)^{\gamma_D} + \frac{b_{weak}}{\gamma_D} g^2 \left[\left(\frac{m_0}{\mu}\right)^{\gamma_D} - 1\right]. \end{aligned} \quad (41)$$

Specializing to the SUSY initial conditions of Eq. (36), that is $\Delta\tilde{g}_0^2, \Delta g_{D0}^2 \approx 0$, and running down to the mass gap of the strong sector, Λ_{comp} , we arrive at

$$\begin{aligned}\frac{\Delta\tilde{g}^2}{g^2} &= b_{weak} g^2 \ln(m_0/\Lambda_{comp}) \\ \frac{\Delta g_D^2}{g^2} &= \frac{b_{weak}}{\gamma_D} g^2 \left[\left(\frac{m_0}{\Lambda_{comp}} \right)^{\gamma_D} - 1 \right].\end{aligned}\tag{42}$$

This then translates into non-cancelling gauge radiative corrections to composite scalars,

$$\Delta m_\phi^2 \sim \frac{\Delta g^2}{16\pi^2} \Lambda_{comp}^2.\tag{43}$$

Thus, despite gauge interactions connecting the strongly coupled matter and weakly coupled fermions over a large hierarchy, it would appear that a high degree of SUSY is accidentally maintained in the light composites, permitting a substantial hierarchy between Λ_{comp} and the mass of light composite scalars. But again, it is interesting that this hierarchy cannot naturally be too large. This is our extension of the mechanism of Partial SUSY. We shall make more quantitative illustrative estimates in Section 7.

6 Non-SUSY Warped Models from the CFT View

Let us rapidly review the core structure of realistic weakly-coupled non-SUSY models in warped 5D spacetime that exploit the RS1 mechanism to solve the hierarchy problem. This will pave the way for incorporating partial SUSY and understanding what it implies for the real world. In particular, we will follow Ref. [30] as our non-SUSY prototype. It developed out of the suggestion that the basic RS1 warped hierarchy mechanism could operate with most of the SM fields propagating in the 5D bulk spacetime [31], that flavor hierarchies from extra-dimensional wavefunction overlaps [32] could arise attractively in the warped setting [33] [34] [35], and out of studies of the implications of the host of precision experimental tests [36].

We use AdS/CFT duality to map to purely 4D models of Higgs compositeness. In the dual description, the extra dimension is replaced by a strongly interacting sector with a large- N_{CFT} expansion, which is approximately conformal above a scale Λ_{comp} . The KK excitations of the higher-dimensional description correspond to “meson” and “glueball” composites, with weak couplings between each other, set by $1/N_{CFT}$. The Kaluza-Klein scale, m_{KK} , characteristic of the KK spectrum is thereby identified with the strong dynamics mass gap, Λ_{comp} , and is taken to be several TeV. The warp factor effects in 5D translate into strong RG effects. The hierarchy problem is solved by the Higgs boson being a light composite of the strong dynamics, captured in 5D by Higgs localization at the IR boundary.

Of course, the 5D models are non-renormalizable effective field theories. Nevertheless they can quantitatively correlate a number of observables. But they rely on the existence of an appropriate UV completion. In purely 4D terms they require the existence of CFTs with all but a finite set of (minimal CFT-color singlet) primary operators having large scaling

dimensions. The finite set of low-dimension operators are the key components in coupling to particles external to the CFT.

To start the story, we can consider all the particles of the SM, with the exception of the Higgs, as being elementary particles outside of the strong CFT sector. They couple (dominantly) linearly to the CFT:

$$\mathcal{L}(m_0) = \mathcal{L}_{CFT} + \mathcal{L}_{SM-H} + A_\mu J_{CFT}^\mu + \bar{\Psi}_i \psi_{Li}. \quad (44)$$

Here, we are specifying the theory at some fundamental UV scale, which we will call m_0 for later convenience in Section 7. The SM gauge bosons, A_μ , couple to global symmetry currents of the CFT sector (thereby “gauging” them), while the chiral SM fermions, ψ_L (in all left-handed notation), couple gauge-invariantly to fermionic composite operators of the CFT, Ψ [37]. At lower scales, μ , we get

$$\mathcal{L}(\mu) = \mathcal{L}_{CFT} + \mathcal{L}_{SM-H} + A_\mu J_{CFT}^\mu + \left(\frac{\mu}{M}\right)^{\gamma_i} \bar{\Psi}_i \psi_{Li}, \quad (45)$$

where the Ψ have some CFT scaling dimensions which we write as $5/2 + \gamma_i$. (The J_{CFT} , being conserved currents have dimension 3.) At $\mu = \Lambda_{comp}$ we match onto SM effective field theory. (We will suppress discussion of the generation of this scale by the dual of the non-SUSY Goldberger-Wise mechanism [25] [6] [7].) The couplings of elementary particles to CFT operators match onto couplings to the Higgs composite:

$$\mathcal{L}_{SM}(\mu < \Lambda_{comp}) = \mathcal{L}_{SM-H} + |D_\mu H|^2 - V(H) + \left(\frac{\Lambda_{comp}}{M}\right)^{\gamma_i} \left(\frac{\Lambda_{comp}}{M}\right)^{\gamma_j} Y_{ij} \psi_{Li} \psi_{Lj} H. \quad (46)$$

Here, the Y_{ij} represent non-hierarchical matrix elements of the strong dynamics. As can be seen, hierarchies in SM Yukawa couplings nevertheless emerge from the RG factors, $(\Lambda_{comp}/M)^{\gamma_i}$.

The case of $\gamma_i < 0$ is subtle, but necessary at least for fitting the large top quark Yukawa coupling. In this case the corresponding CFT coupling to an elementary SM fermion is relevant in the IR and can become strong just below m_0 . This drives the original CFT to a new CFT, in which that SM fermion is realized as a light “meson” [38], in a manner similar to Seiberg duality. In this way, some of the heavier SM fermions can, like the Higgs, be thought of as light composites of a strong CFT, and only the remaining lighter fermions and SM gauge fields are external to the CFT.

Positive γ_i clearly corresponds to irrelevant couplings of the CFT to SM fermions in the IR, and to small SM Yukawa couplings. But here we must have $\gamma_i < 1$ in order that hierarchies among Yukawa coupling are considerably smaller than the overall hierarchy m_0/Λ_{comp} . This explains the need for strong coupling in the 4D description, not just in the IR at Λ_{comp} , but all the way up to m_0 . At weak coupling, we can trust canonical power-counting, and the only composite operator with scaling dimension in the vicinity of $5/2$ is the product of a scalar and a fermion. But in the absence of supersymmetry, the presence of a weakly coupled scalar field in the CFT sector would be unnatural.

Warped models of the type described are in a sense highly successful and attractive in correlating a great deal of qualitative information, in particular the appearance of the

electroweak hierarchy and Yukawa or flavor hierarchies. This is correlated with a suppression of new physics contamination in the lightest SM particles, which happen to be the best tested. In particular it alleviates the most stringent constraints of compositeness tests, electroweak precision tests, flavor-changing neutral currents and CP violation. The case of the electroweak T -parameter is somewhat exceptional, in that satisfying experimental constraints is not automatic but requires custodial isospin to be taken as an approximate accidental symmetry of the CFT sector [30].

But the hierarchy problem is not perfectly solved, precision data still require the KK mass scale (Λ_{comp}) to be several TeV⁶ which acts as the cutoff of the effective low-energy SM. The hierarchy between this scale and the weak scale continues to pose a non-trivial “Little Hierarchy Problem”. This is just where the improved partial SUSY may help, with striking phenomenological consequences.

7 Aspects of Partial SUSY and Realism

We are deferring until Ref. [18] a full and phenomenologically sound 5D warped model, dual to the principles discussed in this paper. Nevertheless we would like to provide enough of a sketch to suggest the viability and interest of the scenario. In particular we want to estimate how much of a little hierarchy between the weak scale and Λ_{comp} is natural with partial SUSY.

7.1 Basic qualitative features

In the scenario of Section 5 applied to the real world, SM gauge fields are accompanied by gauginos, the weakly-coupled fermions ψ_{weak} (with corresponding $\gamma_i > 0$) are accompanied by sfermions, and the strong CFT is really a strong SCFT whose composites, the Higgs and some of the heavy SM fermions, also come in complete supermultiplets, Φ . The linear couplings of ψ_{weak} to SCFT operators are (slightly) irrelevant and therefore, while they are important for the generation of hierarchical effective SM Yukawa couplings, they are subdominant from the viewpoint of SUSY breaking corrections to the Higgs mass. As long as $\Phi = t_R, (t_L, b_L), H_u, H_d$ are all composite chiral supermultiplets of the SCFT, the top Yukawa coupling can arise from the effective composite superpotential of Eq. (30) with minimal impact on Higgs mass. For simplicity here we will take the ψ_{weak} to consist of just the first two generations and the Φ to consist of the Higgs multiplets and the entire third generation. (This will leave the smallness of m_b, m_τ unexplained, but we will do a more careful job in Ref. [18].)

We must also outfit our theory in the UV with a protective symmetry against the appearance of terms linear in the D^a associated to the abelian hypercharge group. Yet the MSSM itself does not possess such a discrete or continuous symmetry. For example, charge conjugation invariance is broken in a chiral theory such as the MSSM. We could assume that the theory down to Λ_{comp} is charge conjugation invariant, in particular the ψ_{weak} coming

⁶In particular, the array of CP-violation tests provide the most stringent constraints [39] [40].

in non-chiral representations. Let us denote these as ψ_{weak} , which are our first two generations of chiral SM fermions, and ψ_{weak}^c which are exotic charge conjugate partners for them. Charge conjugation can then be spontaneously broken at Λ_{comp} , below which it is no longer needed. The unwanted ψ_{weak}^c could then acquire Dirac masses with some of exotic chiral composites of the strong dynamics. But there is a phenomenological difficulty with such a proposal. Dirac masses of this type would be suppressed by the RG running of the ψ_{weak} couplings to the CFT sector,

$$m_{Dirac} \sim \left(\frac{\Lambda_{comp}}{m_0}\right)^{\gamma_i} \Lambda_{comp}. \quad (47)$$

The smallness of the electron Yukawa $\sim 10^{-5}$ is supposed to be explained by a product of two such running factors, so at least one of these factors must be small, $(\frac{\Lambda}{m_0})^{\gamma_i} \leq 1/300$. Even $\Lambda_{comp} \sim 10$ TeV would then result in a charged exotic lepton with a mass of only 10's of GeV, which is ruled out experimentally. A safe model-building rule is therefore to choose a protective symmetry which does not require the ψ_{weak} to contain charged exotic particles.

A simple way of doing this is to extend the SM gauge group to a fully non-abelian gauge group, but in a way in which the SM fermions still fill out complete multiplets. Most obviously one might consider $SU(5)$ as the UV gauge group, spontaneously broken to the SM at Λ_{comp} . This symmetry clearly forbids any term linear in D^a in the effective theory above Λ_{comp} . But it brings many phenomenological difficulties in its wake, rapid proton decay being the worst. A better choice is Pati-Salam unification with gauge group $SU(4) \times SU(2)_L \times SU(2)_R$, Higgsed at Λ_{comp} by the strong dynamics down to $SU(3) \times SU(2)_L \times U(1)_Y$. SM generations are in complete multiplets of the UV gauge group (if we include exotic SM-sterile neutrinos) and so no exotic charged fermions are required or present a conflict with past particle searches. We will choose to proceed with a slightly more minimal choice of UV symmetry, $SU(3) \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, augmented by the discrete “left-right” symmetry under which one exchanges the $SU(2)_L$ and $SU(2)_R$ quantum numbers and simultaneously charge conjugates with respect to $SU(3) \times U(1)_{B-L}$ [41]. The discrete symmetry protects against D_{B-L} terms in the effective theory above Λ_{comp} , and SM generations (again including SM-sterile exotic neutrinos) fill out complete representations.

Until now we have not discussed an origin for gaugino masses protected by R -symmetry. We do not want them to be massless or we would already have seen them. But we do need them to be light enough to play a role in safeguarding composite scalar masses. The most natural possibility is that the Λ_{comp} scale at which conformality is broken in the strong sector is accompanied by a moderately lower threshold at which accidental SUSY is broken in the strong sector, and communicated by \tilde{g} to give gaugino masses. Again, the modelling of this within warped compactifications will be deferred to Ref. [18].

The issue of custodial isospin symmetry in the Higgs sector and our experimental measure of violations given by the electroweak T -parameter is different in partial SUSY compared with completely non-SUSY theories [19]. The non-SUSY case is compatible with a single SM-like Higgs multiplet, so that in particular the Higgs VEV automatically preserves custodial isospin, even as it break electroweak symmetry. In the partial SUSY case, the Higgs sector is minimally MSSM-like and involves two Higgs multiplets. Custodial isospin is generally

violated for unequal Higgs VEVs. This poses an extra danger for the T -parameter. But this danger is offset by the higher compositeness scale, Λ_{comp} , that naturally arises in partial SUSY. The higher scale suppresses new physics contributions to T . We will study the issue more carefully in Ref. [18].

A final important phenomenological consideration, which we will also defer to Ref. [18] is precision gauge coupling unification, based on a variation of the result of Ref. [42] appropriate to partial SUSY.

7.2 Numerical Estimates

Let us see how natural a Higgs mass of say 200 GeV is if $\Lambda_{comp} \sim 10$ TeV. Such a high Λ_{comp} gives a relatively safe suppression of virtual composite effects on the host of precision compositeness, electroweak, flavor and CP tests. The leading corrections come from the $SU(2)_L$ gauge corrections and from top-stop loops. The latter in turn depends on the SUSY breaking stop mass due to $SU(3)$ gauge corrections. We therefore specialize to the case of external gauge group $SU(n = 2, 3)$ and first consider leading radiative corrections to composite scalar masses. These are dominated by Δg_D^2 since it grows fastest in the IR due to the slight relevance of $D^a D^a$, captured by the exponent $\gamma_D > 0$. In the IR effective theory Δg_D^2 gives the SUSY breaking correction to the D-term quartic interaction of composite scalars. At one-loop order, this hard breaking gives rise to an uncancelled quadratic divergence, cut off only by compositeness,

$$\Delta m_\phi^2 = \frac{n^2 - 1}{2n} \frac{\Delta g_D^2}{16\pi^2} \Lambda_{comp}^2. \quad (48)$$

Plugging in Eq. (42) for Δg_D^2 we get

$$\Delta m_\phi^2 = \frac{n^2 - 1}{2n} \frac{g^4}{16\pi^2} \frac{b_{weak}}{\gamma_D} \left[\left(\frac{m_0}{\Lambda_{comp}} \right)^{\gamma_D} - 1 \right] \Lambda_{comp}^2. \quad (49)$$

Here is an illustrative set of numbers. Let us suppose that $m_0 \sim \sqrt{V_0}/M = 10^4$ TeV gives the scale of first two generation sfermion masses. They are so heavy that no precise flavor-degeneracy is needed to be consistent with bounds on their virtual contributions to flavor-changing neutral currents. This situation deserves the name Split SUSY, and without strong interaction miracles this splitting would give extremely large Higgs mass corrections at two-gauge-loop order. The last parameter we need to specify is b_{CFT} for $SU(n = 2, 3)$. We choose $b_{CFT}^{(n=2)} = 1/5$, $b_{CFT}^{(n=3)} = 1/10$. (Recall that the b_{CFT} 's are formally of order $N_{CFT}/(16\pi^2)$ in large- N_{CFT} counting.) We take the gauge couplings at scales of several TeV to be $g_2 \approx 0.6$, $g_3 \approx 1$. Using the leading-in- N_{CFT} running as just a crude estimate we find that $g_{20} \approx 1$, $g_{30} \approx 2$, far from Landau poles. Using Eq. (23) we get

$$\begin{aligned} \gamma_D^{(2)} &\approx 1/12 \\ \gamma_D^{(3)} &\approx 1/4. \end{aligned} \quad (50)$$

It is straightforward to check that for two SM generations of ψ_{weak} ,

$$b_{weak}^{(2)} = b_{weak}^{(3)} = \frac{1}{3\pi^2}. \quad (51)$$

Plugging into Eq. (49), we find SUSY-breaking stop mass corrections

$$\Delta m_{stop}^2 \approx (700\text{GeV})^2. \quad (52)$$

This naturally allows stop masses ~ 700 GeV, light enough not to destabilize a 200 GeV Higgs via top/stop loops. Similarly, we find SUSY-breaking electroweak Higgs mass corrections,

$$\Delta m_H^2 \approx (130\text{GeV})^2. \quad (53)$$

This is also perfectly compatible with a natural Higgs mass of 200 GeV. Thus all the dominant SUSY breaking radiative corrections to Higgs mass are consistent with a naturally light Higgs boson, despite having a 10 TeV compositeness scale.

Let us check that supergravity effects can be unimportant. The anomaly-mediated SUSY breaking contributions to gaugino masses can be smaller than 100 GeV for $M < 10^{14}$ GeV, as discussed in Section 3, taking them out of our consideration. Again from Section 3, the gravitino mass is $\sim \sqrt{V_0}/M_{Pl}$, which is \sim TeV with our numbers, so that it does not constitute the LSP.

8 Collider Implications

Even without a fully detailed model, the outlines of the expected phenomenology can be given. Well below Λ_{comp} we have the MSSM but without the sfermion partners of the light fermions. They have “split off” at the very high scale m_0 . The LHC phenomenology is therefore similar to the scenario of “More Minimal SUSY” [17], even though the UV physics is quite different. At a hadron collider, one will dominantly pair produce the colored gluinos and stops among the new particles. Assuming effective R -parity, these will decay via real or virtual stops into electroweak superpartners, (winos, binos, Higgsinos) plus top or bottom quarks. If the LSP is a neutralino, such events will be seen as top or bottom pairs plus missing energy. Given sufficiently heavy gravitino as discussed above, the LSP may well be a neutralino combination of winos, binos and Higgsinos. This can make a suitable WIMP dark matter candidate in standard fashion. In more detail, it may also be possible to measure the deviations of gaugino couplings from gauge couplings, $\Delta\tilde{g}$, that reflect the highly “split” nature of the spectrum.

The implementation of the protective symmetry against SCFT deformations linear in the D^a , naturally requires an extension of the SM gauge group. This would lead to new W' , Z' s below the Λ_{comp} scale which can be sought at the LHC, such as the new gauge bosons of our $SU(2)_R \times U(1)_{B-L}$ extension of the gauge group. Note these are elementary particles not composites of the strong sector, in particular their couplings to the SM particles are comparable to standard electroweak gauge couplings.

Most spectacularly, there are also composite states of the strong sector, AdS/CFT dual to KK excitations of the SUSY SM in the language of warped compactification, but their mass scale Λ_{comp} is expected to be at several TeV. Some of these may be visible at the LHC if we are lucky. They would certainly dominate the physics of the next generation of high energy colliders.

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